EXPERIMENTAL AND NUMERICAL STUDY ON DYNAMIC BEHAVIOR OF COMPOSITE BEAMS WITH DIFFERENT CROSS SECTION

A thesis submitted in partial fulfillment of
the requirements for the award of

Master of Technology
In
Structural Engineering

By:

Miss Meera

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NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA

Department of Civil Engineering

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Under the guidance of

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CERTIFICATE

This is to certify that the thesis entitled, **"Experimental and Numerical Studies on Dynamic behavior of Composite Beams with different Cross Section"** submitted by **Miss Meera** in partial fulfillment of the requirements for the award of Master of Technology Degree in Civil Engineering with specialization in "Structural Engineering" at National Institute of Technology, Rourkela is an authentic work carried out by her under my supervision and guidance. To the best of my knowledge, the matter embodied in this thesis has not been submitted to any other university/ institute for award of any Degree or Diploma.

She has taken keen and genuine interest in acquiring knowledge and presentation of data in an organized manner.

She is very sincere and hard working towards her work and would provide herself as an asset to the organization that employes her.

I wish her all success in her future endeavor.

Prof. Shishir Kumar Sahu
Dept. of Civil Engineering
National Institute of Technology, Rourkela

Dedicated

to

MY BELOVED PARENTS

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“Although we know there's much to fear, we were moving mountains long before we knew we could…..now I'm standing here, my heart's so full I can't explain.” The words were out from my heart when I was going to write this part. I cannot help giving special recognitions to the following colleagues and friends as angles God sent out to help me moved this mountain and warmed me throughout the journey.

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ABSTRACT

The increasing use of composite materials across various fields such as aerospace, automotive, civil, naval and other high performance engineering applications are due to their light weight, high specific strength and stiffness, excellent thermal characteristic, ease in fabrication and other significant attributes. The present study deals with experimental investigation on free vibration of laminated composite beam and compared with the numerical predictions using finite element method (FEM) in ANSYS environment. A program is also developed in MATLAB environment to study effects of different parameters. The scope of the present work is to investigate and understand the effect of different parameters including cross sectional shape on modal parameters like modal frequency, mode shapes.

Experimental investigation is carried out by Impulsive frequency response test under fixed- free and fixed-fixed boundary conditions. Composites Beams are fabricated using woven glass fabric and epoxy by hand layup technique. Modal analysis of various cross sectional beams were reported, compared and discussed. The finite element modeling has been done by using ANSYS 14 and compared with the experimental results. Two-node, finite elements of three degrees of freedom per node and rectangular section are presented for the free vibration analysis of the laminated composite beams in this work.

The effects of different parameters including ply orientation, number of layers, effect of the length of the beam and various boundary conditions of the laminated composite beams are discussed.

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INTRODUCTION

INTRODUCTION

1.1 Introduction

The widespread use of composite structures in aerospace applications has stimulated many researchers to study various aspects of their structural behavior. These materials are particularly widely used in situations where a large strength-to-weight ratio is required. Similarly to isotropic materials, composite materials are subjected to various types of damage, mostly cracks and delamination. These result in local changes of the stiffness of elements for such materials and consequently their dynamic characteristics are altered. This problem is well understood in case of constructing elements made of isotropic materials, while data concerning the influence of fatigue cracks on the dynamics of composite elements are scarce in the available literature.

This chapter contains a general introduction of the research that was carried out within the framework of this thesis. The research context is described in Sec.2.1. The focus of the thesis as well as the main objectives is discussed in Sec.2.2.

1.2 Research Context

The beam is manufactured from a Glass Fibre Reinforced Polymer (GFRP) and its box and Channel like beam. This beam was actually used as a prototype for footbridge. The GFRP (Glass Fibre Reinforced Polymer) composite materials are being utilized in more structures like bridges as the technology. GFRP composite are ideal for structural applications where high strength to weight and stiffness to weight ratios are needed. As the technology progresses, the cost involved in manufacturing and designing composite material will reduce, thus bringing added cost benefits also.

The vibration analyses in composite beams have been a problem for structural designer for years and have increased recently. Though, all elements have natural frequencies with the potential to suffer excessive vibrations under dynamic load. This is done by using modal analysis, which allows one to determine the natural frequencies of the structure, associated mode shapes and damping. And once natural frequencies are known, thus making structure suitable for the task designed for.

This is mainly due to the human feeling of vibration while crossing a footbridge with a frequency close to the first (fundamental) natural frequency of the bridge, although the vibration caused by the pedestrians are far from harmful to the bridge. Therefore vibration analysis of such structure can be considered to be a serviceability issue. Modal parameters of a structure are frequency, mode shape and damping. Frequency is directly proportional to structure's stiffness and inverse of mass.

Nevertheless, modal parameters are functions of physical properties of the structure. Thus, changes in the physical properties such as, beginning of local cracks and/or loosening of a connection will cause detectable changes in the modal properties by reducing the structure's stiffness.

The design of GFRP (Glass Fibre Reinforced Polymer) bridge deck established to promote the use of innovative material and lead to use in footbridge construction to improve the reliability for proper safety and serviceability. The Aberfeldy cable-stayed was the first GFRP footbridge, built back in 1992. There are only two GFRP footbridge existing in UK knows as the Halgavor and Willcot. The use of glass or carbon fibre reinforced polymer was due to its advantages, for they are easily drawn into having a high strength-to-weight ratio, low maintenance and lightweight.

In this research, Finite element models for different boundary conditions are constructed using the commercial finite element software package ANSYS to support and verify the dynamic measurements. Initial FRP (Fibre Reinforced Polymer) composite channel section and box section beams were created. Furthermore, the FEM results of the beams are compared to the experimental solution for understanding of the relationship between the FE results. The natural frequencies and mode shapes of the composite beam are obtained after performing modal analysis which the author contribution to this area of research.

1.3 Purpose and Objectives of Study

The main objective of this thesis is to study and compare the numerical and experimental result of free vibration analysis of composite Fibre Reinforced Polymer (FRP) beam. The present investigation mainly focuses on the study of vibration of industry driven woven fiber glass/epoxy composite beams. A first order shear deformation theory based on finite element model is developed for studying the free vibration, The influence of shape of the beams, boundary conditions, number of layers, fiber orientations and aspect ratio on the free vibration of composite beams are investigated experimentally also examined numerically.

1.4 Organization of Thesis

This thesis contains six chapters. In Chapter 1, a brief introduction of the importance of the study has been outlined.

In Chapter 2, a detailed review of the literature pertinent to the previous research works made in this field has been listed. A critical discussion of the earlier investigations is done. The aim and scope of the present study are also outlined in this chapter.

In Chapter 3, a description of the theory and formulation of the problem and the finite element procedure used to analyse the vibration of composite beams is explained in detail. The computer program used to implement the formulation is briefly described.

In Chapter 4, the experimental investigation for free vibration and static stability of different shaped composite beam, are described in detail. This chapter includes fabrication procedures for samples, test set-up, apparatus required for different tests and determination of material constants.

In Chapter 5, the 3D finite Element Modelling with the help of ANSYS for free vibration and static stability of different shaped composite beam, are described in detail. This chapter includes procedures of modeling.

In Chapter 6, the results and discussions obtained in the study have been presented in detail. The natural frequency of composite beams, are studied experimentally; the influence of boundary conditions, number of layers, shape, and aspect ratio on the free vibration of channel section and box section composite beams are investigated experimentally and numerically.

In Chapter 7, the conclusions drawn from the above studies are described. There is also a brief note on the scope for further investigation in this field.

In Chapter 8, Some important publications and books referred during the present investigation have been listed in the **References**.

Chapter 2

LITERATURE REVIEW

LITERATURE REVIEWS

2.1 Introduction

The widespread use of composite structures in aerospace applications has stimulated many researchers to study various aspects of their structural behavior. These materials are particularly widely used in situations where a large strength-to-weight ratio is required. Similarly to isotropic materials, composite materials are subjected to various types of damage, mostly cracks and delamination. These result in local changes of the stiffness of elements for such materials and consequently their dynamic characteristics are altered. Therefore a comprehensive understanding of the behavior is of fundamental importance in the assessment of structural performance of laminated composites. Thus the dynamic characteristics are of great technical importance for understanding the dynamic systems under periodic loads. Though the present investigation is mainly focused on free vibration of composite beams, some relevant researches on free vibration and static stability or buckling of beams are also studied for the sake of its relevance and completeness.

2.2 Review on vibration of composite beam

Nikpour & Dimarogonas (1988) presented the local compliance matrix for unidirectional composite materials. They have shown that the interlocking deflection modes are enhanced as a function of the degree of anisotropy in composites.

Ostachowicz & **Krawczuk** (1991) presented a method of analysis of the effect of two open cracks upon the frequencies of the natural flexural vibrations in a cantilever beam. Two types of cracks were considered: double-sided, occurring in the case of cyclic loadings, and single-sided, which in principle occur as a result of fluctuating loadings. It was also assumed that the cracks occur in the first mode of fracture: i.e., the opening mode. An algorithm and a numerical example were included.

Krawczuk (1994) formulated a new beam finite element with a single non-propagating one-edge open crack located in its mid-length for the static and dynamic analysis of cracked composite beam-like structures. The element includes two degrees of freedom at each of the three nodes: a transverse deflection and an independent rotation respectively. He presented the exemplary numerical calculations illustrating variations in the static deformations and a fundamental bending natural frequency of a composite cantilever beam caused by a single crack.

Krawczuk & **Ostachowicz** (1995) investigated eigen frequencies of a cantilever beam made from graphite-fiber reinforced polyimide, with a transverse on-edge non-propagating open crack. Two models of the beam were presented. In the first model the crack was modeled by a massless substitute spring Castigliano‟s theorem. The second model was based on the finite element method. The undamaged parts of the beam were modeled by beam finite elements with three nodes and three degrees of freedom at the node. The damaged part of the beam was replaced by the cracked beam finite element with degrees of freedom identical to those of the non-cracked done. The effects of various parameters the crack location, the crack depth, the volume fraction of fibers and the fibers orientation upon the changes of the natural frequencies of the beam were studied. Computation results indicated that the decrease of the natural frequencies not only depends on the position of the crack and its depth as in the case of isotropic material but also that these changes strongly depend on the volume fraction of the fiibers and the angle of the fibers of the composite material.

Ghoneam (1995) presented the dynamic characteristics laminated composite beams (LCB) with various fiber orientations and different boundary fixations and discussed in the absence and presence of cracks. A mathematical model was developed, and experimental analysis was utilized to study the effects of different crack depths and locations, boundary conditions, and various code numbers of laminates on the dynamic characteristics of CLCB. The analysis showed good agreement between experimental and theoretical results.

Kisa (2003), investigated the effects of cracks on the dynamical characteristics of a cantilever composite beam, made of graphite fibre-reinforced polyamide. The finite element and the component-mode synthesis methods were used to model the problem. The cantilever composite beam divided into several components from the crack sections. The effects of the location and depth of the cracks, and the volume fraction and orientation of the fibers on the natural frequencies and mode shapes of the beam with transverse non-propagating open cracks, were explored. The results of the study leaded to conclusions that, presented method was adequate for the vibration analysis of cracked cantilever composite beams, and by using the drop in the natural frequencies and the change in the mode shapes, the presence and nature of cracks in a structure can be detected.

Li Jun, Hua Hongxing(2008) presented the exact dynamic stiffness matrix of a uniform laminated composite beam based on trigonometric shear deformation theory. The dynamic stiffness matrix is formulated directly in an exact sense by solving the governing differential equations of motion that describe the deformations of laminated beams according to the trigonometric shear deformation theory, which includes the sinusoidal variation of the axial displacement over the cross-section. The derived dynamic stiffness matrix is then used in conjunction with the Wittrick–Williams algorithm to compute the natural frequencies and mode shapes of the composite beams.

Volkan Kahya(2011) studied on a multilayered shear deformable beam element for dynamic analysis of laminated composite beams subjected to moving loads. The laminated beam element includes separate rotational degrees of freedom for each lamina while it does not require any additional axial and transversal degrees of freedom beyond those necessary for a single lamina. The shape functions are selected to ensure compatibility and continuity between the laminae. Interracial slip and domination are not allowed. Results are given for moving load-induced vibrations of laminated composite beams. Effects of the load speed, boundary conditions, and laminate lay-up on the beam response is studied.

Jeong et al.(1995) investigated experimentally the dynamic characteristics of high strength symmetrically laminated carbon fiber epoxy composite thin beams were in a vacuum chamber equipped with a fiber optic vibrometer and the electromagnetic hammer. It was found that the macromechanical theory could accurately predict the dynamic characteristics of the carbon fiber epoxy composite thin beams when the undirectional properties of the composite material were known.

Gil Lee et al. (1998) investigated in order to improve the damping capacity of the column of a percision mirror surface grinding machine tool, a hybrid column was manufactured by adhesively bonding glass fiber reinforced epoxy composite plates to a cast iron column. To optimize the damping capacity of the hybrid column, the damping capacity of the hybrid column was calculated with respect to the fiber orientation and thickness of the composite laminate plate and compared to the measured damping capacity. From experiments, it was found that the damping capacity of the hybrid column was 35% higher than that of the cast iron column.

Jaehong Lee(2000) investigated on free vibration analysis of a laminated beam with delaminations is presented using a layerwise theory. Equations of motion are derived from the Hamilton's principle, and a finite element method is developed to formulate the problem. Numerical results are obtained addressing the effects of the lamination angle, location, size and number of delamination on vibration frequencies of delaminated beams. It is found that a layerwise approach is adequate for vibration analysis of delaminated composites.

Lee and Kim (2002) developed a general analytical model applicable to the dynamic behavior of a thin-walled I-section composite. This model is based on the classical lamination theory, and accounts for the coupling of fexural and torsional modes for arbitrary laminate stacking sequence configuration, i.e. unsymmetric as well as symmetric, and various boundary conditions. A displacement-based one-dimensional finite element model is developed to predict the natural frequencies and corresponding vibration modes for a thin walled composite beam. Equations of motion are derived from Hamilton's principle. Numerical results are obtained for thin-walled composites addressing the effects of fiber angle, modulus ratio, height-to-thickness ratio, and boundary conditions on the vibration frequencies and mode shapes of the composites.

Lee and Kim (2002) developed a general analytical model applicable to the dynamic behavior of a thin-walled channel section composite. This model is based on the classical lamination theory, and accounts for the coupling of fexural and torsional modes for arbitrary laminate stacking sequence configuration, i.e. unsymmetric as well as symmetric, and various boundary conditions. A displacement-based one-dimensional finite element model is developed to predict the natural frequencies and corresponding vibration modes for a thin walled composite beam. Equations of motion are derived from Hamilton's principle. Numerical results are obtained for thin-walled composites addressing the effects of fiber angle, modulus ratio, height-to-thickness ratio, and boundary conditions on the vibration frequencies and mode shapes of the composites.

He and Zhu (2011) investigated on Fillets which are commonly found in thin-walled beams. Ignoring the presence of a fillet in a finite element (FE) model of a thin-walled beam can significantly change the natural frequencies and mode shapes of the structure. A large number of solid elements are required to accurately represent the shape and the stiffness of a fillet in a FE model, which makes the size of the FE model unnecessarily large for global dynamic and static analyses. The natural frequencies and mode shapes of a thin-walled L-shaped beam specimen calculated using the new methodology are compared with its experimental results for 28 modes. The maximum error between the calculated and measured natural frequencies for all the modes is less than 2% and the associated modal assurance criterion values are all above 95%.

Senthamaraikanan and Raman(2012) investigated on vibration characteristic of carbon epoxy composite beams await increased attention due to their successful usage in structural industries. The effect of cross sectional shape on modal parameters like modal frequency, modal shapes and damping behavior under Fixed-Free, Fixed-Fixed boundary condition.

2.6 Critical discussion

The present review indicates that more studies are conducted on laminated composite plate, beams and shells. However studies involving vibration studies of composite laminates are very limited. As regards to methodology, the researchers are more interested to use numerical methods instead of analytical methods. With the advent of high speed computers, more studies are made using finite element method. From the present review of literature, the lacunae of the earlier investigations which need further attention of future researchers.

2.4 Scope of Present Study

An extensive review of the literature shows that a lot of work was done on the vibration and static stability of delaminated composite beams. The woven composite is a new class of textile composite and has many industrial applications. Very little work has been done on vibration analysis of composite beams with different cross section. The present study is mainly aimed at filling some of the lacunae that exist in the proper understanding of the vibration analysis of industry driven woven fiber beams. Based on the review of literature, the different problems identified for the present investigated for free vibration of composite beams with different cross section. The present study mainly focuses on the parametric resonance characteristics of homogeneous composite beams. The influence of various parameters such as aspect ratio, number of layers, effect of boundary condition, effect of shapes on composite beams are examined experimentally and numerically using ANSYS and finite element method.

Chapter 3

MATHEMATICAL FORMULATIONS

MATHEMATICAL FORMULATIONS

3.1 Introduction

This chapter represents the theory and finite element formulation (FEM) for free vibration, static and dynamic analysis of the composite beam of different cross sections. The basic configuration of the problem investigated here is a composite beam of any boundary condition, however, a typical cantilever composite beam, which has tremendous applications in aerospace structures and high-speed turbine machinery, is considered.

3.2 The Methodology

The governing equations for the vibration analysis of the composite beam are developed. The stiffness matrix and mass matrix of composite beam element is obtained by Krawczuk & Ostachowicz (1995).

The assumptions made in the analysis are:

i. The analysis is linear. This implies constitutive relations in generalized Hook‟s law for the materials are linear.

ii. The Euler–Bernoulli beam model is assumed.

iii. The damping has not been considered in this study.

3.3 Governing Equation

The differential equation of the bending of a beam with a mid-plane symmetry (B_{ij} = 0) so that there is no bending-stretching coupling and no transverse shear deformation (Ɛxz= 0) is given by;

$$IS_{11}\frac{d^4\omega}{dx^4} = q \cdot x$$

It can easily be shown that under these conditions if the beam involves only a one layer, isotropic material, then $IS_{11} = EI = Ebh^3/12$ and for a beam of rectangular cross-section Poisson‟s ratio effects are ignored in beam theory, which is in the line with Vinson & Sierakowski (1991).

In Equation 1, it is seen that the imposed static load is written as a force per unit length. For dynamic loading, if Alembert‟s Principle are used then one can add a term to Equation.1 equal to the product mass and acceleration per unit length. In that case Equation.1 becomes

$$IS_{11}\frac{d^4\omega}{dx^4} = q\;x,\,t - \rho A\frac{\partial^2\omega(x,t)}{\partial x^2}$$

Where ω and q both become functions of time as well as space, and derivatives therefore become partial derivatives, ρ is the mass density of the beam material, and here A is the beam cross-sectional area. In the above, q(*x, t*) is now the spatially varying time-dependent forcing function causing the dynamic response, and could be anything from a harmonic oscillation to an intense one-time impact.

For a composite beam in which different lamina have differing mass densities, then in the above equations use, for a beam of rectangular cross-section,

$$\rho A = \rho bh = \underset{k=1}{N} \rho b\; h\; - h_{k-1}$$

However, natural frequencies for the beam occur as functions of the material properties and the geometry and hence are not affected by the forcing functions; therefore, for this study let q(x, t) be zero.

Thus, the natural vibration equation of a mid-plane symmetrical composite beam is given by;

$$IS_{11}\frac{d^4\omega}{dx^4} + \rho\,A\frac{\partial^2\omega(x,t)}{\partial x^2} = 0$$

It is handy to know the natural frequencies of beams for various practical boundary conditions in order to insure that no recurring forcing functions are close to any of the natural frequencies, because that would result almost certainly in a structural failure. In each case below, the natural frequency in radians/unit time are given as

$$\omega_n = \alpha^2 \left(\frac{IS_{11}}{\rho A L^4}\right)^1 2$$

Where is the co-efficient, which value is catalogued by Warburton, Young and Felgar and Once is known then the natural frequency in cycles per second (Hertz) is given by f_n = ω_n /2π, which is in the line with Vinson & Sierakowski (1991).

3.4 Mathematical Modeling

The model chosen is a cantilever composite beam of uniform cross-section A, The width, length and height of the beam are B, L and H, respectively in Figure.3.1. The angle between the fibers and the axis of the beam is ‘α’.

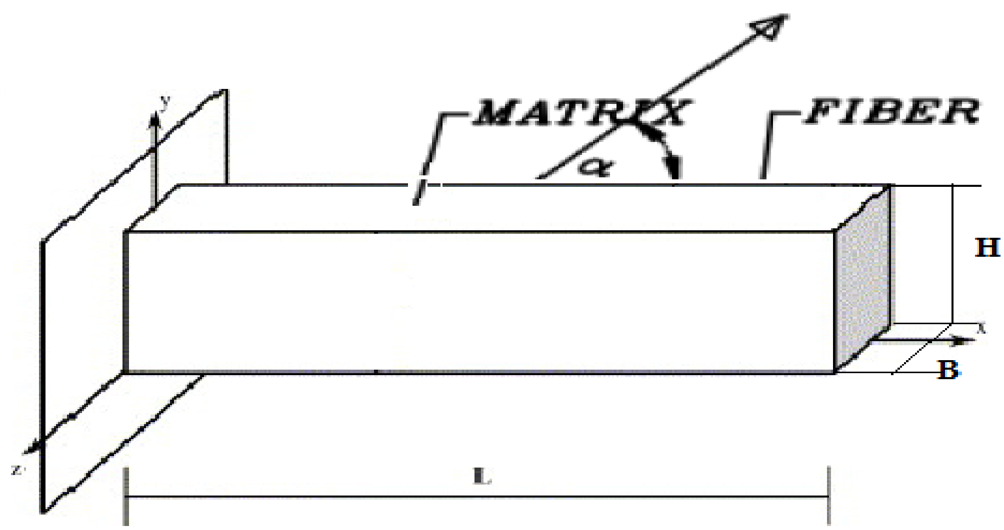


Figure 3.1 Schematic diagram cantilever composite beam

3.4.1 Vibration Study Analysis

Mass and stiffness matrices of each beam element are used to form global mass and stiffness matrices. The dynamic response of a beam for a conservative system can be formulated by means of Lagrange‟s equation of motion in which the external forces are expressed in terms of time-dependent potentials and then performing the required operations the entire system leads to the governing matrix equation of motion

$$M\, q + K_e - P(t) K_g \; q = 0$$

where "q" is the vector of degree of freedoms. M, K_e, K_g are the mass, elastic stiffness and geometric stiffness matrices of the beam. The periodic axial force $P\ t = P_o + P_t Cos\,\Omega t$, where Ω is the disturbing frequency, the static and time dependent components of the load can be represented as a fraction of the fundamental static buckling load Pcr hence putting $P t = \alpha P_{cr} + \beta P_{cr}\ Cos\,\Omega t$.

In this analysis, the computed static bucking load of the composite beam is considered the reference load. Further the above equation reduces to other problems as follows.

i. Free vibration with α = 0, β = 0 and ω = Ω/2 the natural frequency

$$K_e - \omega^2 M \;\; q = 0$$

ii. Static stability with α = 1, β = 0, Ω = 0

$$K_e - P_{cr} K_g \;\; q = 0$$

3.4.2 Derivation of Element Matrices

In the present analysis two noded composite beam element with three degree of freedom (the axial displacement, transverse displacement and the independent rotation) per node is considered. The characteristic matrices of the composite beam element are computed on the basis of the model proposed by Oral (1991). The stiffness and mass matrices are developed from the procedure given by Krawczuk & Ostachowicz (1995).

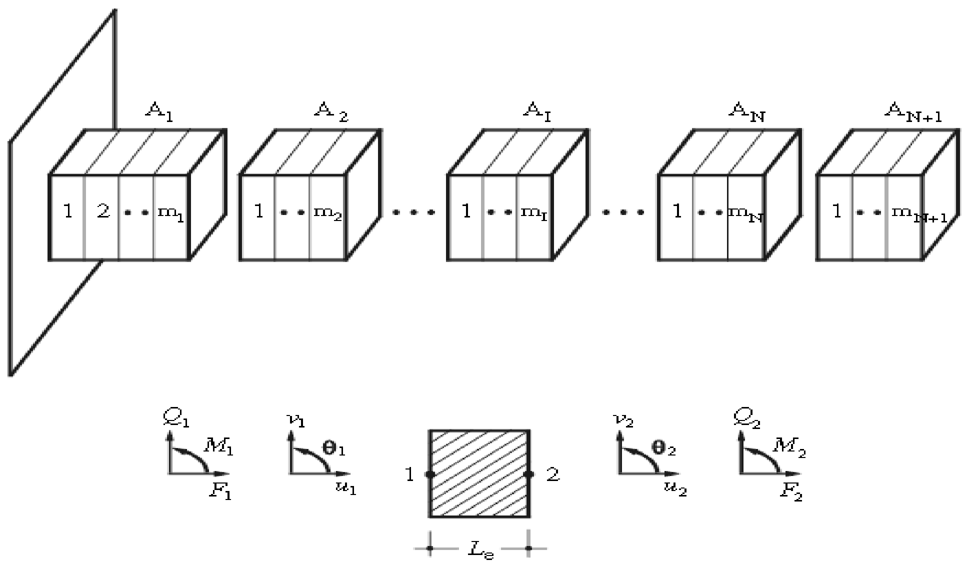


Figure 3.2 Components of the composite beam and dividing them into the finite number of elements.

The linear strain can be described in terms of displacements as

$$\varepsilon = \mathbf{B}\,\partial$$

where displacement vector in the element reference beam is given as

$$\partial = [u_1\, v_1\, \theta_1\, u_2\, v_2\, \theta_2]^T$$

3.4.3 Stress-Strain matrix

$$D-\begin{bmatrix} S_{11} & S_{13} \\ S_{13} & S_{33} \end{bmatrix}$$

where the element of the matrix D are expressed in Appendix A.

Following standard procedures the element stiffness matrix, mass matrix and geometrical stiffness matrix can be expressed as follows:

3.4.4 Element stiffness matrix

Element stiffness matrix for a three-nodes composite beam element with three degrees of freedom δ = (u, v, θ) at each node, for the case of bending in the x, y plan, are given in the line Krawczuk & Ostachowicz (1995) as follows:

$$K_e = IS_{11} \,_v [B]^T D\, B\, dv$$

where [B] = $\frac{\partial}{\partial x^2} N =$ strain displacement matrix, [N] = shape function matrix and

$K_e = \lfloor K_{ij} \rfloor_{6\times 6}$ where $\lfloor K_{ij} \rfloor_{6\times 6}$ = *(i= j= 1, 2……..6)* are

$$k_{11} = k_{55} = 7BHS_{33}/3L_e,$$

$$k_{12} = k_{21} = -k_{56} = -k_{65} \ = BHS_{33}/2$$

$$k_{13} = k_{31} = k_{35} = k_{53} \ = -8BHS_{33}/3L$$

$$k_{14} = k_{41} = k_{36} = k_{63} = -k_{23} = -k_{32} = -k_{45} = -k_{54} = -2BHS_{33}/3$$

$$k_{15} = k_{51} = BHS_{33}/3L_e,$$

$$k_{16} = k_{61} = -k_{25} = -k_{52} = -BHS_{33}/6,$$

$$k_{22} = k_{66} = BH(7H^2 S_{11}/36L + L S_{33}/9)$$

$$k_{24} = k_{42} = k_{46} = k_{64} = BH(-2H^2 S_{11}/9L + L S_{33}/9)$$

$$k_{26} = k_{62} = BH(H^2 S_{11}/36L + L S_{33}/18)$$

$$k_{33} = -16BHS_{33}/3L_e,$$

$$k_{44} = BH(4H^2 S_{11}/9L + 4L S_{33}/9)$$

$$k_{34} = k_{43} = 0$$

where B is the width of the element, H is the height of the element and L denotes the length of the element. S_{11}, S_{13}, and S_{33} are the stress-strain constants.

3.4.5 Generalized element mass matrix

Element mass matrix of the non-cracked composite beam element is given in the line Krawczuk & Ostachowicz (1995) as

$$K_e = \rho\ {}_v[N]^T\ N\,dv$$

$M_e = \lfloor M_{ij} \rfloor_{6\times6}$ where $\lfloor M_{ij} \rfloor_{6\times6} =$ *(i= j= 1, 2…….6)* are

$$m_{11} = m_{55} = 2\rho BHL_e/15,$$

$$m_{12} = m_{21} = -m_{56} = -m_{65} = \rho BHL^2/180,$$

$$m_{13} = m_{31} = m_{35} = m_{53} = \rho BHL_e/15,$$

$$m_{14} = m_{41} = -m_{45} = -m_{54} = -\rho BHL^2 / 90$$

$$m_{15} = m_{51} = -\rho BHL / 30$$

$$m_{16} = m_{61} = -m_{25} = -m_{52} = \rho BHL^2 / 180,$$

$$m_{22} = m_{66} = \rho BHL(L^2 / 1890 - H^2 / 360),$$

$$m_{24} = m_{42} = m_{46} = m_{64} = \rho BHL(-L^2 / 945 + H^2 / 180),$$

$$m_{26} = m_{62} = \rho BHL(L^2 / 1890 - H^2 / 360),$$

$$m_{33} = 8\rho BHL / 15,$$

$$m_{44} = \rho BHL(2L^2 / 945 + 2H^2 / 45),$$

$$m_{34} = m_{43} = m_{36} = m_{63} = m_{23} = m_{32} = 0,$$

where ρ is the mass density of the element, B is the width of the element, H is the height of the element and L denotes the length of the element.

3.5 Flow Chart of Program

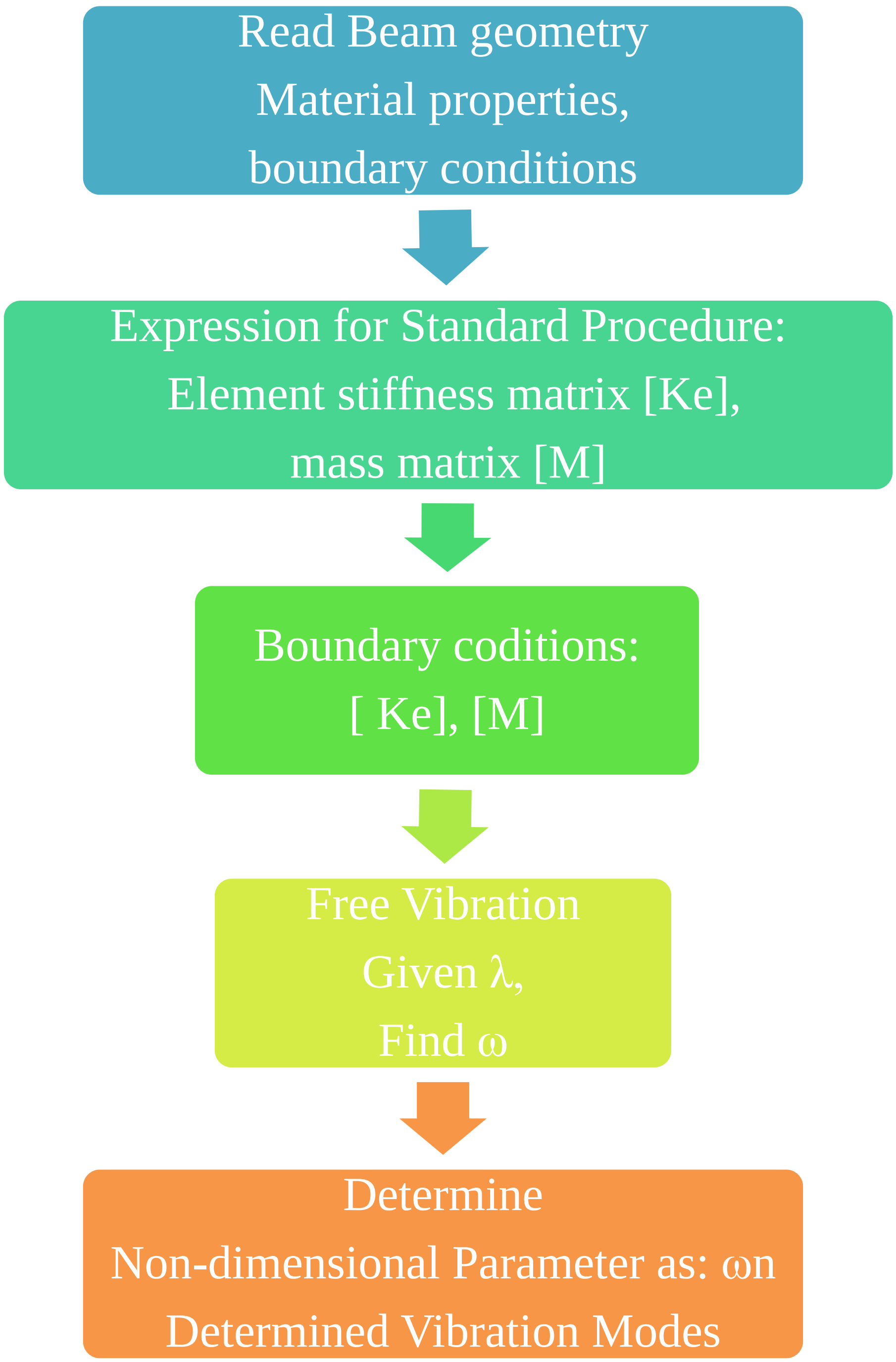


Fig.3.5 Flowchart utilized in Free vibration analysis

Chapter 4

EXPERIMENTAL PROGRAMME

EXPERIMENTAL PROGRAMME

4.1 Introduction

This chapter deals with the details of the experimental works conducted on the static analysis and free vibration of industry driven woven roving composite beams. Therefore composite beams are fabricated for the aforementioned experimental work and the material properties are found out by tensile test as per ASTM D3039/ D3039M (2008) guidelines to characterize the composite beams. The experimental results are compared with the analytical or numerical predictions. The experimental work performed is categorized in three sections as follows:

- **Static analysis**
- **Determination of material constants**
- **Vibration study**

4.2 Experimental program for static analysis

4.2.1 Materials

- The following constituent materials were used for fabricating the laminate:
- Woven roving glass fiber as reinforcement
- Epoxy as resin
- Hardener
- Polyvinyl alcohol as a releasing agent

4.2.2 Fabrication of specimens

In the present investigation, the glass epoxy laminate was fabricated in a proportion of 50:50 by weight fractions of fiber matrix. Araldite LY-556, an unmodified epoxy resin based on

Bisphenol-A and hardener (Ciba-Geig, India) HY-951, aliphatic primary amine were used with woven roving E-glass fibers treated with silane based sizing system (Saint-Gobain Vetrotex) to fabricate the laminated composite beam. Woven roving glass fibers were cut into required shape and size for fabrication. Epoxy resin matrix was prepared by using 10% hardeners. Contact moulding in an open mould by hand lay-up was used to combine plies of woven roving (WR) in the prescribed sequence. A flat plywood rigid platform was selected. A plastic sheet i.e. a mould releasing sheet was kept on the plywood platform and a thin film of polyvinyl alcohol was applied as a releasing agent. Laminating starts with the application of a gel coat (epoxy and hardener) deposited on the mould by brush, whose main purpose was to provide a smooth external surface and to protect the fibers from direct exposure to the environment. Subsequent plies were placed one upon another with the matrix in each layer to obtain sixteen stacking plies. The laminate consisted of 8 layers of identically 0-90^0 oriented woven fibers as per ASTM D2344/ D2344M (2006) specifications. The mould and lay up were covered with a release film to prevent the lay up from bonding with the mould surface. Then the resin impregnated fibers were placed in the mould for curing. The laminates were cured at normal room temperature under a pressure of 0.2 MPa for three days. After proper curing of the composite beams, the release films were detached. From the laminates the specimens were cut for three-point bend test (Figure 4.1a & 4.1b) by brick cutting machine into 45 x 6mm (Length Breadth) size as per ASTM D2344/ D2344 specification and the thickness was taken as per the actual measurement. The average thickness of specimens for bend test is 3.0 mm.

4.2.3 Bending test

The most commonly used test for ILSS is the short beam strength (SBS) test under three point bending. The SBS test was done as per ASTM D 2344/ D 2344 M (2006) by using the INSTRON 1195 material testing machine. The specimens were tested at 2, 50, 100, 200 and 500 mm/minute cross head velocities with a constant span of 34 mm to obtain interlaminar shear strength (ILSS) of samples. Before testing, the thickness and width of the specimens were measured accurately. The test specimen was placed on the test fixtures and aligned so that its midpoint was centered and its long axis was perpendicular to the loading nose. The load was applied to the specimen at a specified cross head velocity. Breaking load of the sample was recorded. About five samples were tested at each level of experiment and their average value along with standard deviation (SD) and coefficient of variation (CV) were reported in result part. The interlaminar shear strength was calculated using the formula,

S = (0.75*Pb*)/*bd* as per ASTM D 2344

Where *Pb* is the breaking load in kg; *b* is the width in mm and *d* is the thickness in mm.

Figure 4.1 (a): Three point bend test setup and fixture

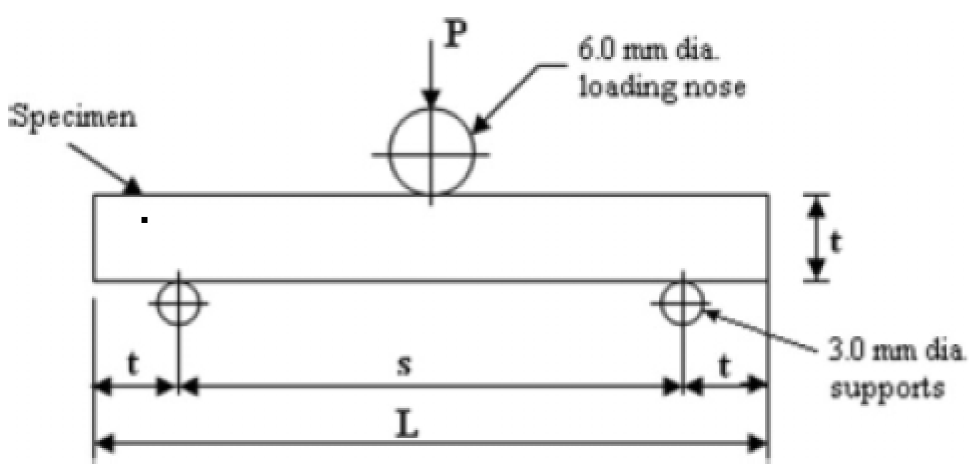


Figure 4.1 (b): Schematic diagram of three point bend test

4.3 Determination of material constants

Laminated composite plates behave like orthotropic lamina, the characteristics of which can be defined completely by four material constants i.e. E1, E2, G12, and V12 where the suffixes 1 and 2 indicate principal material directions. For material characterization of composites, laminate having eight layers was fabricated to evaluate the material constants. The constants are determined experimentally by performing unidirectional tensile tests on specimens cut in longitudinal and transverse directions, and at 45° to the longitudinal direction, as described in ASTM standard: D 3039/D 3039 M (2008). The tensile test specimens are having a constant rectangular cross section in all the cases. The dimensions of the specimen are mentioned below in Table 4.1.

Table 4.1: Size of the specimen for tensile test

Length(mm) Width(mm) Thickness(mm)	Width(mm)	Thickness(mm)
200	25	3

The specimens were cut from the plates themselves by diamond cutter or by hex saw as per requirement as shown in Figure 4.2 (a). Four replicate sample specimens were tested and mean values were adopted. The test specimens are shown in Figure 4.2. (b) to Figure 4.2(d).

Figure 4.2(a)

Figure 4.2(b)

Figure 4.2(c)

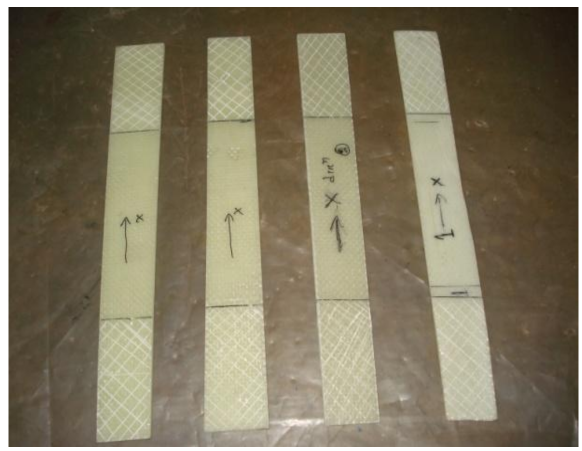


Figure 4.2(d)

Figure 4.2(a): Diamond cutter for cutting specimens, (b) Specimens in Y direction, (c) Specimens in 45 direction, (d) Specimens in X direction.

Coupons were machined carefully to minimize any residual stresses after they were cut from the plate and the minor variations in dimensions of different specimens are carefully measured. For measuring the Young's modulus, the specimen was loaded in INSTRON 1195 universal testing machine (as shown in Figure 4.3) monotonically to failure with a recommended rate of extension (rate of loading) of 0.2 mm/minute. Specimens were fixed in the upper jaw first and then gripped in the movable jaw (lower jaw). Gripping of the specimen should be as much as possible to prevent the slippage. Here, it was taken as 50mm in each side for gripping. Initially strain was kept at zero. The load, as well as the extension, was recorded digitally with the help of a load cell and an extensometer respectively. Failure pattern of woven fiber glass/epoxy composite specimen is shown in Figure 4.4. From these data, engineering stress vs. strain curve was plotted; the initial slope of which gives the Young's modulus. The ratio of transverse to longitudinal strain directly gives the Poisson's ratio by using two strain gauges in longitudinal and transverse direction. But here Poisson.s ratio is taken as 0.3.

The shear modulus was determined using the following formula from Jones [1975] as:

$$G_{12} = \frac{1}{\frac{4}{E_{45}} - \frac{1}{E_1} - \frac{1}{E_2} + \frac{2\vartheta_{12}}{E_1}}$$

The values of material constants finally obtained experimentally for vibration are presented in Chapter-6.

Figure 4.3: Tensile test of woven fiber glass/epoxy composite specimens

Figure 4.4: Failure pattern of woven fiber glass/epoxy composite specimen

4.4 Experimental programme for vibration study

4.4.1 Fabrication of specimens

The fabrication procedure for preparation of the composite beams of channel section and box section in case of vibration study was bit difficult. Artificial metal moulds were fabricated to maintain the shape. Specimens are fabricated by hand layup technique and cured under room temperature. The laminate consisted of eight layers of identically 0- 90° oriented woven fibers to maintain thickness of the beam as 5mm. Beams are fabricated by maintaining constant moment of inertia and uniform cross sectional area with uniform length of 400mm in order to evaluate the shape Effect. After completion of all the layers, again aplastic sheet was covered on the top of last ply by applying polyvinyl alcohol inside thesheet as releasing agent. Again one flat ply board and a heavy flat metal rigid platformwas kept at the top of the beams for compressing purpose. The plates were left for aminimum of 48 hours before being transported and cut to exact shape for testing. All the specimens are tested for free vibration analysis. The geometrical dimensions (i.e.length, breadth, and thickness), ply orientations of the fabricated beams are shown in Table-4.2. All the specimens described in Table 4.2 were tested for its vibration characteristics. To study the effect of boundary condition on the natural frequency of fabricated beams, the beams were tested for three different boundary conditions (B.C) i.e. for cantilever, Fixed-Fixed, Free-Free. For different boundary conditions, one iron frame was used.

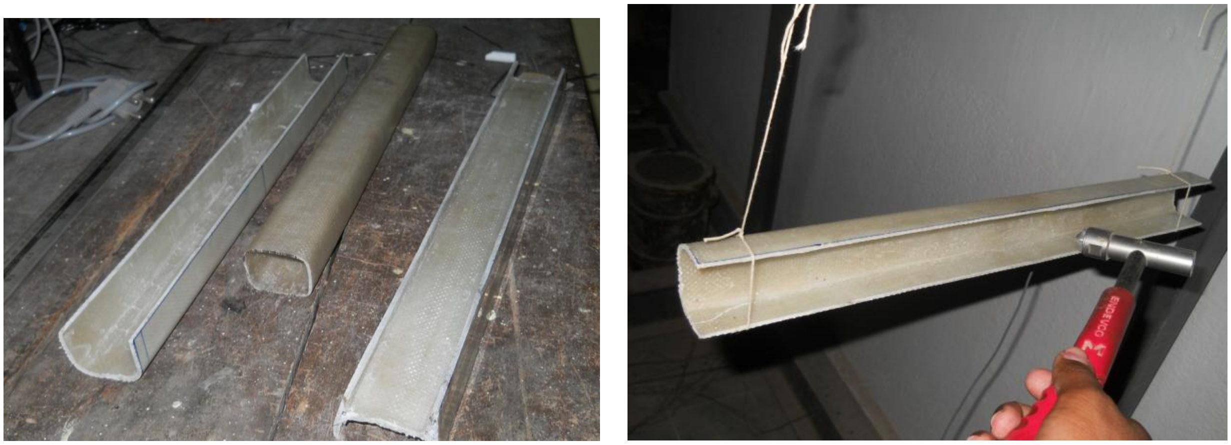


Figure 4.4: glass/epoxy composite specimen fabricated with different shapes

Sections	Height H(m)	Width B(m)	Weight (kg)	Area (m^2)
box	0.04	0.03	0.31	4.5
channel	0.04	0.03	0.32	5.3

Table 4.2 Proporties of composite beam specimen

4.4.2 Equipments for vibration test

In order to achieve the right combination of material properties and service performance, the dynamic behavior is the main point to be considered. To avoid the typical problems caused by vibrations, it is important to determine the natural frequency of the structure and the modal shapes to reinforce the most flexible regions or to locate the right positions where weight should be reduced or damping should be increased. The fundamental frequency is a key parameter. The natural frequencies are sensitive to the orthotropic properties of composite plates and design-tailoring tools may help in controlling this fundamental frequency. Due to the advancement in computer aided data acquisition systems and instrumentation, experimental modal analysis or free vibration analysis has become an extremely important tool in the hands of an experimentalist. The apparatus which are used in free vibration test are

- Modal hammer (type 2302-5)
- Accelerometer (type 4507)
- FFT Analyzer (Bruel Kajer FFT analyzer type .3560)
- Notebook with PULSE software.
- Specimens to be tested

The apparatus which is used in the vibration test are shown in Figure 4.7 to Figure 4.10.

Figure 4.7: Modal Impact Hammer(type 2302-5)

Figure 4.8: Accelerometer (4507)

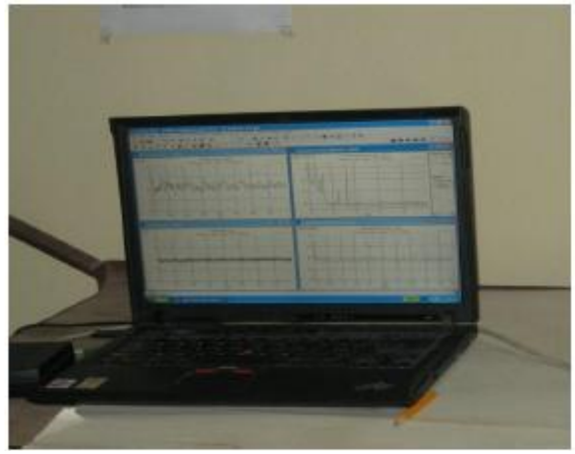


Figure 4.9: Bruel & Kajer FFT analyzer   Figure 4.10: Display unit

4.4.3 Procedure for free vibration test

The setup and the procedure for the free vibration test are described sequentially as given below. The test specimens were fitted properly to the iron frame. The connections of FFT analyzer, laptop, transducers, modal hammer, and cables to the system were done. The pulse lab shop version-10.0 software key was inserted into the port of the computer. The beams were excited in a selected point by means of small impact with an impact hammer (Model 2302-5) for cantilever, Fixed-Fixed and Free-Free boundary condition. The input signals were captured by a force transducer, fixed on the hammer. The resulting vibrations of the specimens on the selected point were sensed by an accelerometer. The accelerometer (B&K, Type 4507) was mounted on the specimen by means of bees wax. The signal was then processed by the FFT Analyzer and the frequency spectrum was also obtained. Both input and output signals are investigated by means of spectrum analyzer (Bruel & kajer) and resulting frequency response functions are transmitted to a computer for modal parameter extraction. The output from the analyzer was displayed on the analyzer screen by using pulse software. Various forms of frequency response functions (FRF) were directly measured. However, the present work represents only the natural frequencies of the beams. For FRF, at each singular point the modal hammer was struck five times and the average value of the response was displayed on the screen of the display unit. At the time of striking with a modal hammer to the points on the specimen precaution were taken for making the stroke to be perpendicular to the surface of the beams. Then by moving the cursor to the peaks of the FRF graph the frequencies are measured.

Chapter 5

MODELING IN ANSYS

MODELING IN ANSYS

5.1 Introduction

The finite element simulation was done by FEA package known as ANSYS. The FEA software package offerings include time-tested, industry-leading applications for structural, thermal, mechanical, computational fluid dynamics, and electromagnetic analyses, as well as solutions for transient impact analysis. ANSYS software solves for the combined effects of multiple forces, accurately modelling combined behaviours resulting from "multiphysics" interactions.

This is used to perform the modelling of the beam and calculation of natural frequencies with relevant mode shapes.This is used to simulate both the linear & nonlinear effects of structural models in a static or dynamic environment. The advanced nonlinear structural analysis includes large strain, numerous nonlinear material models, nonlinear buckling, post-buckling, and general contact. Also includes the ANSYS Parametric Design Language (APDL) for building and controlling user-defined parametric and customized models.

The purpose of the finite element package was utilised to model the Fibre reinforced polymer (FRP) beam in 3-D as SHELL93 (8node93). This package enables the user to investigate the physical and mechanical behavior of the beam.

The FE-model parameters extracted from the *Strongwell* manual [19] provided with the composite beam specimen. The FE-model constructed along the vertical direction only which made it applicable to the real bridge model. The load applied from pedestrian used to come in vertical directions during the walking or movement along the bridge that is why the analysis is being done toward vertical directions. Though, the specimen has anisotropy properties but we have only considered the vertical direction that is why the linear isotropic parameter only used.

5.2 Procedure in Modelling ANSYS

There are major and sub important steps in ANSYS model, pre-processing, solution stage and post-processing stage.

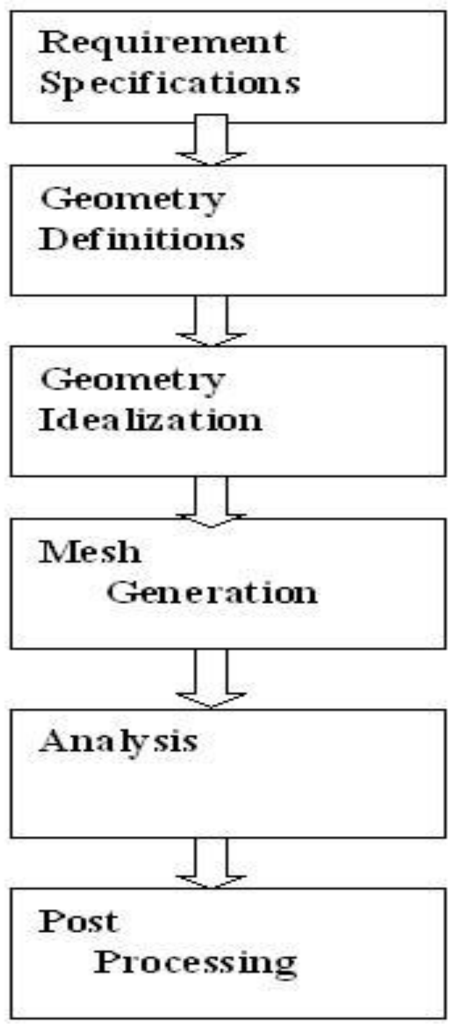


Figure 6.1: FE-Analysis Steps

5.2.1 Requirement Specification

This step is done in pre-processing in ANSYS. In this work the beam element model used know was SHELL93 and it was specification at the pre-processing stage. The SHELL93 element is applicable to this model for the structural meshing and boundary condition applications.

Geometry Definition	Values
Thickness	3.57e-4m
Young modulus	1.46e9
Density	1660
Width	0.05m
Length of the beam	0.40m
Poisson Ratio	0.3

Table 5.1:Input data for Modelling of the beam

The parameter specified in the table above indicated that only vertical direction analysis was carried on the beam. This is applicable to the modal analysis experiment in the previous section.

5.2.2 Idealization Specification

This is sub-stepping procedure in model context represents a 3D shell definition. This model is optimized for rapid FEM analysis and is composed of 2D geometry, beam surface model. It is easy to locate and calculate the numerical position in shell geometry; beam shell model can be defined of the 3D definition. The analysis type is defined as modal

5.2.3 Mesh Generation

The generation of a mesh on the idealized geometry is done through meshed model. The meshing depend on the configuration for the model, the general rules are carried out by setting a density for the mesh. In this application, loads and boundary conditions are added in the input file. The solver input file consists of mesh elements, nodes and load cases. The input file is generated from the application containing mesh elements, nodes and boundary conditions are added to the file.

5.2.4 Analysis

This is a stage where solution was conducted. It was the step to pre-processing and different stages of analysis took place. The load is applied to edges of beam, this was easier to implement in SHELL model. And the other entire complex algorithm in FEM solved.

5.2.5 Post-processing

At this stage the results of analysis are obtained numerically and graphically.

Chapter 6

RESULT & DISCUSSION

RESULT AND DISCUSSION

6.1 Introduction

In this chapter the results obtained from ANSYS 12 software package are used for the numerical results given below, the procedure to obtained the results on ANSYS given on the chapter 5 and Experimental procedure for the experimental results are given in chapter 4. The ANSYS program must be first verified in order to ensure the subsequent analyses are free of error. Therefore the result obtained from the analysis is compared with available results of references. Natural frequencies obtained from experimental and ANSYS are listed in tables and thoseresults comparing with the available results of references for thecomposite laminated beam with different boundary conditions. And mode shapes are presented by graphs for different boundary conditions.

6.2 Comparison with Previous Studies

In order to check the accuracy of the present analysis, the case considered in Kisa (2004) is adopted here for Isotropic beam and the case considered in Li Jun (2008) is adopted for Composite beam.

To find out the natural frequencies and mode shapes of beam, finite element solution program done by ANSYS.

6.2.1 Vibration analysis studies of isotropic beam

Length, L = 0.2m, Breadth , b= 0.0078m

Depth, d= 0.025m, E= 216.19e9 Nm-2

V= 0.28, P = 7.85×10^3.

Cantilever boundary condition considered.

Natural frequencies	Present study	Kisa (1998)
1^{st} Mode	1038.21	1037.01
2^{nd} mode	6506.89	6458.34
3^{rd} mode	18229.11	17960.54
4^{th} mode	35780.05	34995.429

Table 6.1: First three non-dimensional frequencies of isotropic beam

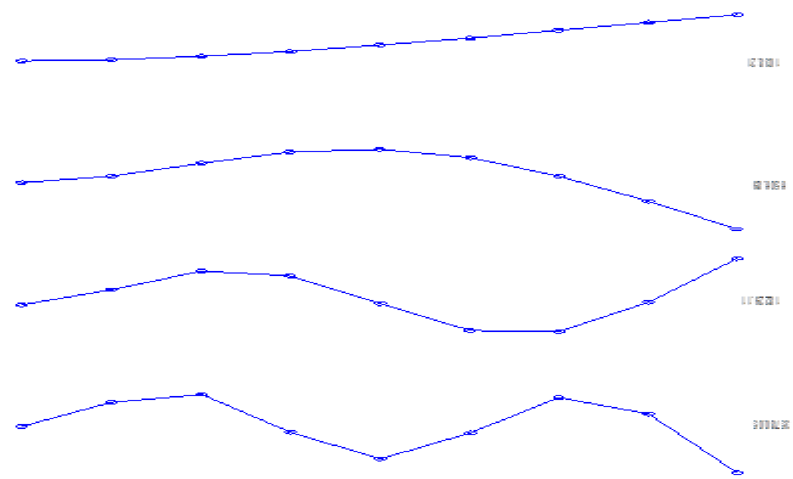


Figure 6.1 : 1^{st} four natural frequency mode shapes

6.2.2 Vibration analysis studies of Composite beam

Modulus of elasticity, E11 =144.80 Pa

E22 = 9.65 Pa

Modulus of rigidity, G12 = 4.14 pa

G13 = 3.45 pa

Poisson's Ratio, ν_{12} = 0.3

Mass density, ρ = 1389.23kg/m3

Length , L = 0.381m,

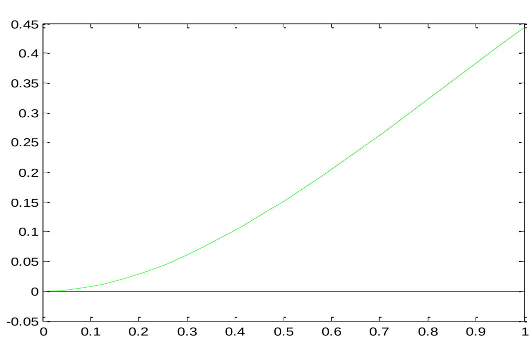
Height, h = 25.4*10^-3 m,

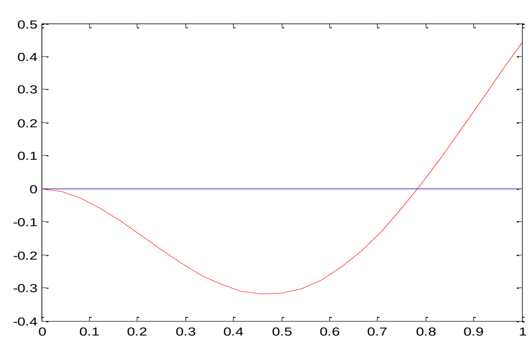
Breadth, b = 25.4*10^-3 m

Both Cantiliver and Fixed boundary condition is considerd

	Clamp -Clamp		Clamp -Free	
Natural frequencies	Present study	Jun,Honhxing (2009)	Present study	Jun,Honhxing (2009)
1st Mode	637.74	638.5	105.37	105.39
2nd mode	1656.41	1657.3	636.71	637.67
3rd mode	3032.19	3034.0	1696.94	1698.0
4th Mode	4663.51	4661.2	2391.11	2392.3
5th Mode	4780.74	4784.6	3119.27	3121.0

Table 6.2 Comparison of Natural frequencies (Hz) of [30/50/30/50] composite beam





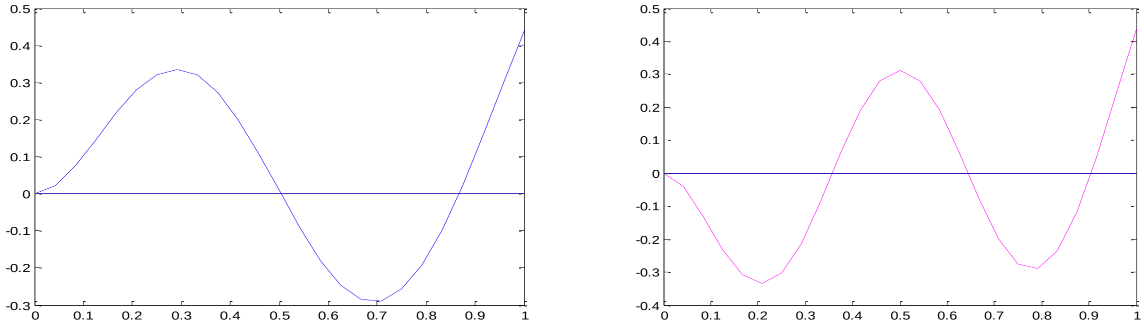


Figure6.2: Four natural frequency mode shapes of composite beams

6.3 Experimental and Numerical Results

Numerical (FEM) and experimental results of frequencies of vibration for [0/0]8s woven fiber Glass/Epoxy composite beams are obtained for different boundary conditions. The boundary conditions considered for the present numerical analysis as well as experimental work are - cantilever, Fixed-Fixed, simply supported.

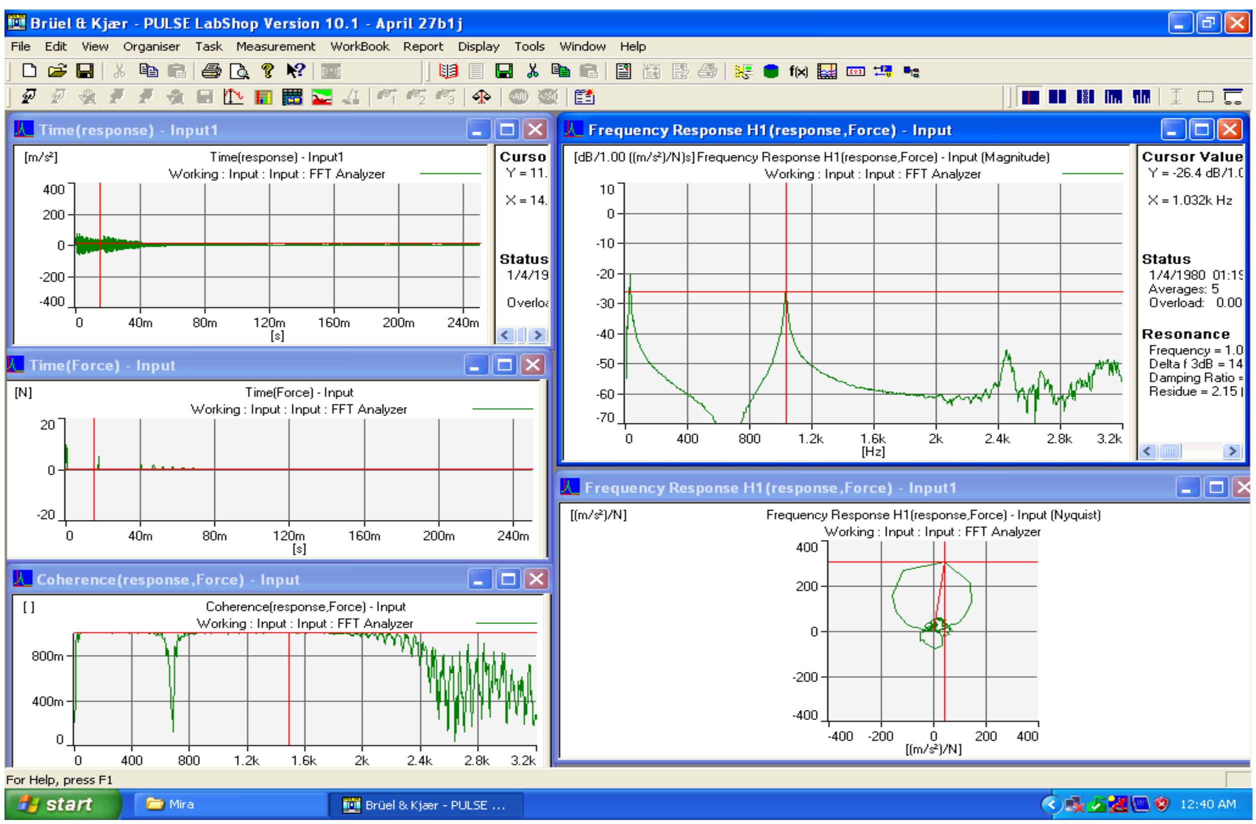


Figure 6.3: The different peaks of FRF shows the different modes of vibrations and the coherence

6.3.1 Fixed-Fixed Boundary Condition

Channel section; fixed-fixed; 8 layers

		ANSYS		
Frequency	Experimental (L=400mm)	L=400mm	L=600mm	L=800mm
ω 1	272	246.05	137.34	87.882
ω 2	436	420.85	193.45	110.11
ω 3	652	647.72	335.91	204.43
ω 4	-	892.77	458.26	275.26
ω 5	-	910.67	521.00	301.41

Table 6.3: First five non-dimensional frequencies of composite channel section of 8 layers with Fixed-Fixed Boundary condition

Channel section; fixed-fixed ; 6 layers

	ANSYS		
Frequency	L=400mm	L=600mm	L=800mm
ω 1	245.68	121.98	
ω 2	415.96	199.41	
ω 3	591.84	305.91	
ω 4	704.77	448.47	
ω 5	767.58	513.23	

Table 6.4: First five non-dimensional frequencies of composite channel section of 6 layers with Fixed-Fixed Boundary condition

Channel section; Fixed-Fixed; 4 layer

	ANSYS		
Frequency	L=400mm	L=600mm	L=800mm
ω 1	225.61	109.58	66.169
ω 2	404.23	187.41	106.71
ω 3	512.27	278.69	168.55
ω 4	514.15	437.52	258.98
ω 5	552.28	480.20	291.45

Table 6.5: First five non-dimensional frequencies of composite channel section of 4 layers with Fixed-Fixed Boundary condition

Box section; fixed-fixed; 8 layers

		ANSYS		
Frequency	Experimental (L=400mm)	L=400mm	L=600mm	L=800mm
ω 1	532.0	526.95	249.23	143.51
ω 2	604.0	593.41	281.12	162.01
ω 3	1187.0	1295.7	652.22	385.53
ω 4	1397.6	1457.9	734.12	434.54
ω 5	1612.0	1598.4	1075.0	732.24

Table 6.6: First five non-dimensional frequencies of composite box section of 8 layers with Fixed-Fixed Boundary condition

Box section; fixed-fixed ; 6 layers

	ANSYS		
Frequency	L=400mm	L=600mm	L=800mm
ω 1	512.81	241.92	139.13
ω 2	582.46	276.30	159.29
ω 3	1258.0	634.13	374.45
ω 4	1420.2	720.09	426.95
ω 5	1461.3	1061.4	711.79

Table 6.7: First five non-dimensional frequencies of composite box section of 6 layers with Fixed-Fixed Boundary condition

Box section; Fixed-fixed; 4 layer

	ANSYS		
Frequency	L=400mm	L=600mm	L=800mm
ω 1	498.64	235.86	135.71
ω 2	566.97	268.12	154.33
ω 3	1152.0	615.59	364.71
ω 4	1200.0	700.39	414.57
ω 5	1379.0	1021.6	690.08

Table 6.8: First five non-dimensional frequencies of composite box section of 4 layers with Fixed-Fixed Boundary condition

6.3.2 Fixed-Free Boundary Condition

Box section; cantilever ; 8 layers

		ANSYS		
Frequency	Experimental (L=400mm)	L=400mm	L=600mm	L=800mm
ω 1	136	91.116	40.801	23.011
ω 2	-	102.90	46.099	26.004
ω 3	532.00	532.62	248.34	142.17
ω 4	657	599.16	279.99	160.45
ω 5	872	806.23	538.19	390.07

Table 6.9: First five non-dimensional frequencies of composite box section of 8 layers with Cantilever Boundary condition

Box section; cantilever ; 6 layers

	ANSYS		
Frequency	L=400mm	L=600mm	L=800mm
ω 1	88.228	39.488	22.267
ω 2	101.19	45.336	25.574
ω 3	517.45	240.83	137.75
ω 4	588.45	275.27	157.79
ω 5	800.43	535.09	378.47

Table 6.10: First five non-dimensional frequencies of composite box section of 6 layers with Cantilever Boundary condition

Box section; cantilever ; 4 layer

	Ansys		
Frequency	L=400mm	L=600mm	L=800mm
ω 1	86.071	35.524	21.723
ω 2	97.900	43.835	24.721
ω 3	503.35	234.84	134.37
ω 4	571.65	266.82	152.77
ω 5	792.97	532.43	368.69

Table 6.11: First five non-dimensional frequencies of composite box section of 4 layers with Cantilever Boundary condition

Channel section; Cantilever; 8 layers

		ANSYS		
Frequency	Experimental (L=400mm)	L=400mm	L=600mm	L=800mm
ω 1	83.00	69.244	30.889	17.397
ω 2	89.00	72.570	39.323	23.940
ω 3	201.0	192.62	99.677	63.739
ω 4	323.0	305.84	171.82	108.58
ω 5	-	419.34	191.31	111.34

Table 6.12: First five non-dimensional frequencies of composite channel section of 8 layers with Cantilever Boundary condition

Channel section;   Cantilever; 6 layers

Frequency	L=400mm	L=600mm
ω 1	63.902	30.565
ω 2	68.521	34.420
ω 3	182.79	89.714
ω 4	275.27	146.20
ω 5	414.52	189.28

Table 6.12: First five non-dimensional frequencies of composite channel section of 6 layers with Cantilever Boundary condition

Channel section;   Cantilever; 4 layer

	ANSYS		
Frequency	L=400mm	L=600mm	L=800mm
ω 1	52.319	28.678	16.857
ω 2	67.101	29.931	18.487
ω 3	172.25	81.862	49.140
ω 4	241.09	123.72	78.737

Table 6.13: First five non-dimensional frequencies of composite channel section of 4 layers with Cantilever Boundary condition

6.4 Analysis Results

4.4.1 Effect of Boundary Condition

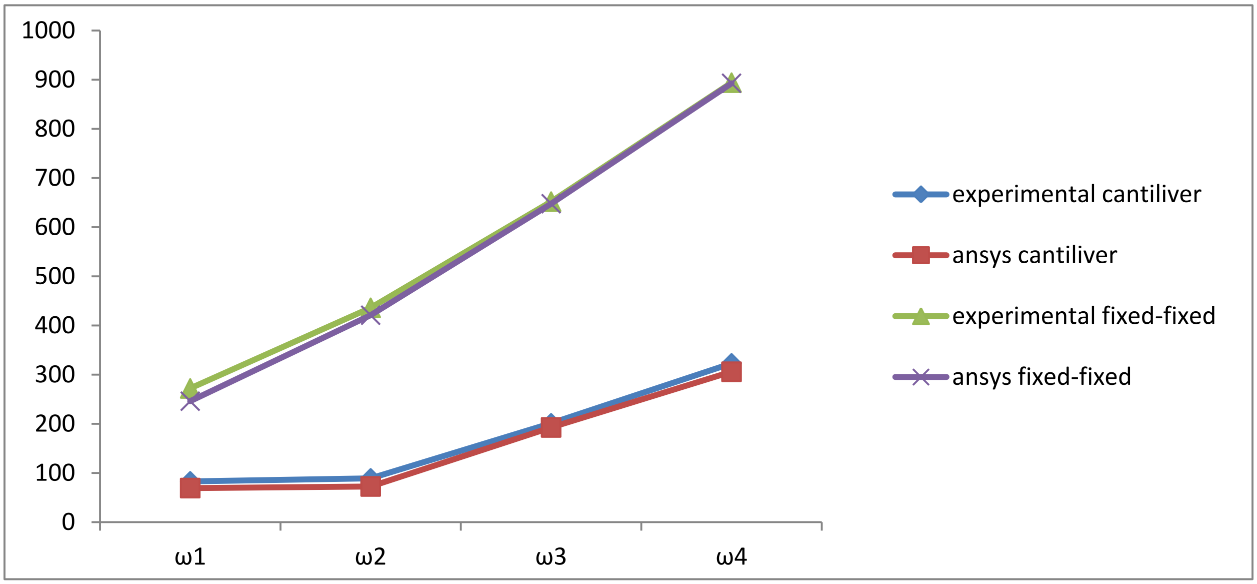


Figure 6.4 The comparison between computational and Experimental results of channel section under different boundary condition. The natural frequencies are much same.

4.4.2 Effect of Layers

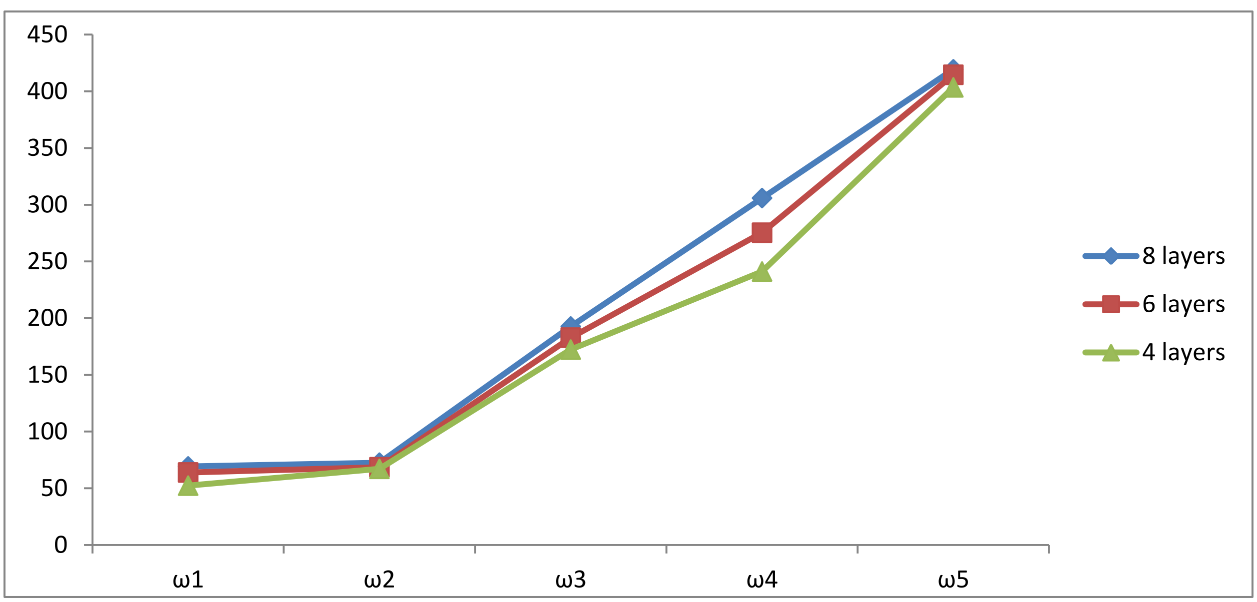


Figure 6.5 Effect of layers on free vibration of a cantilever channel section. The natural frequency increases with increases in no. of layers.

4.4.3 Effect of Length

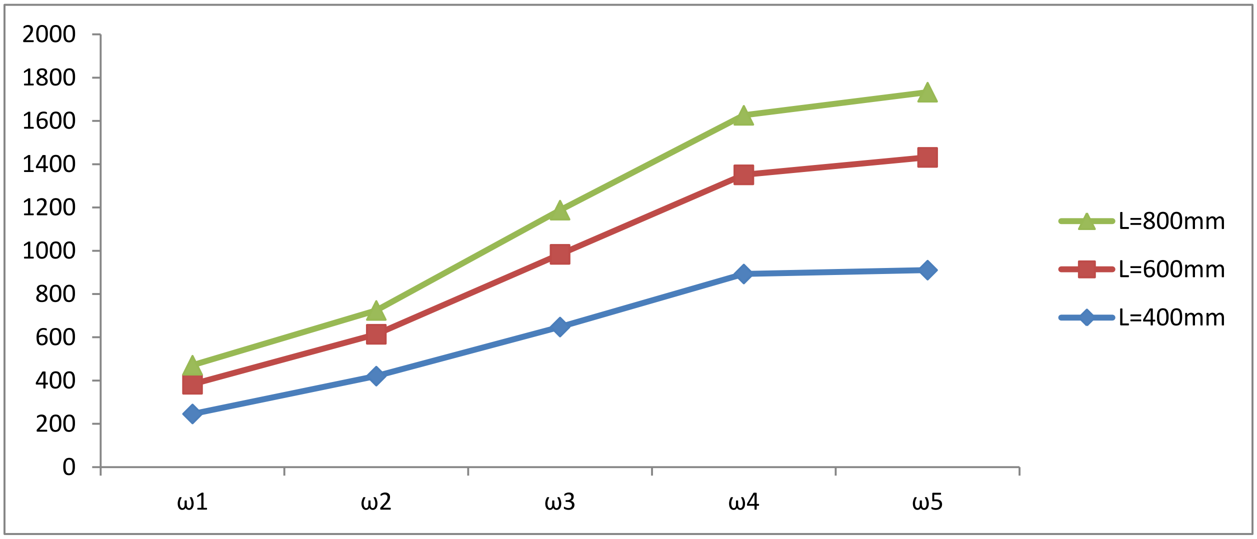


Figure 6.6 Effect of length on free vibration of a Fixed-Fixed channel section. The natural frequency decreases with increases in length of the beam.

4.4.4 Effect of Length

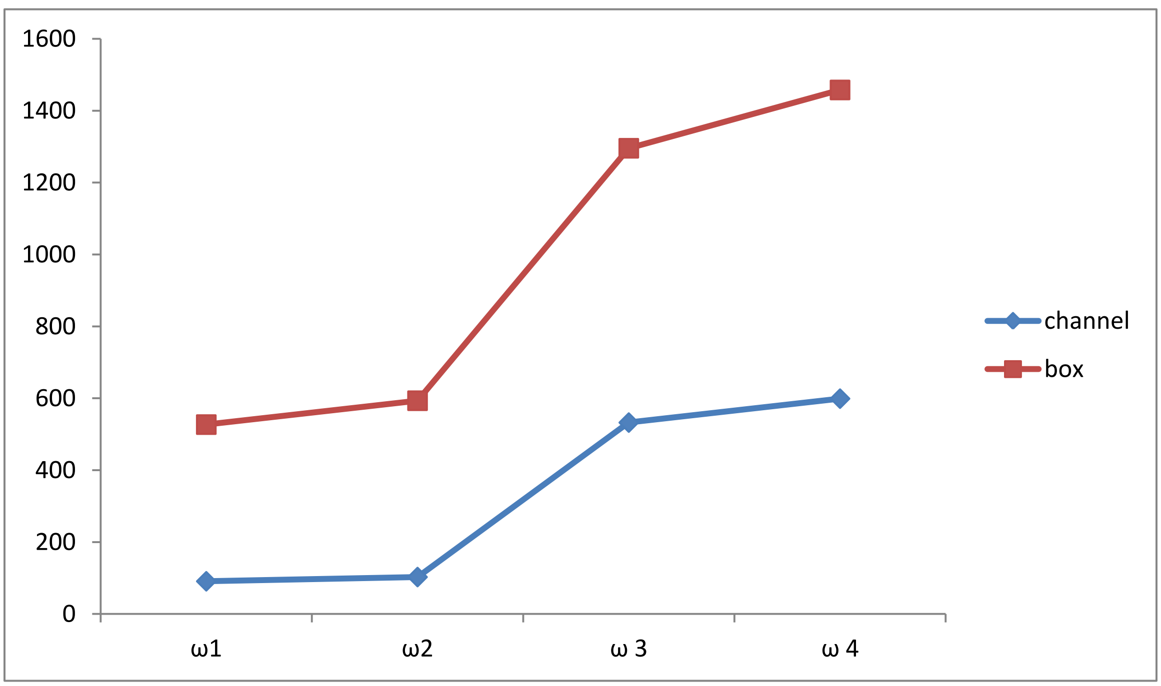


Figure 6.7: effect of Shape on free vibration of a box section. The natural **frequency** is minimum for cantilever and maximum for fixed beam

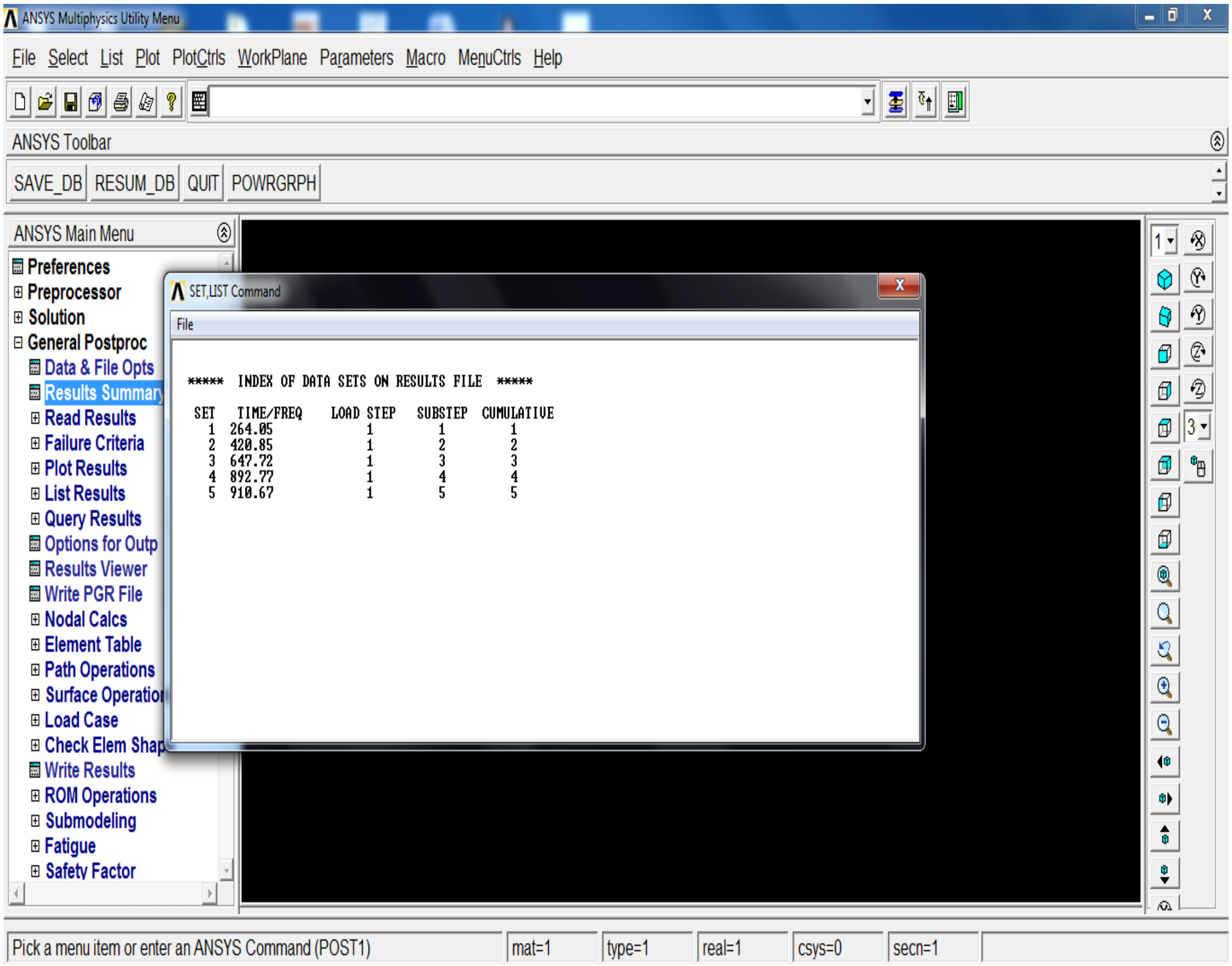
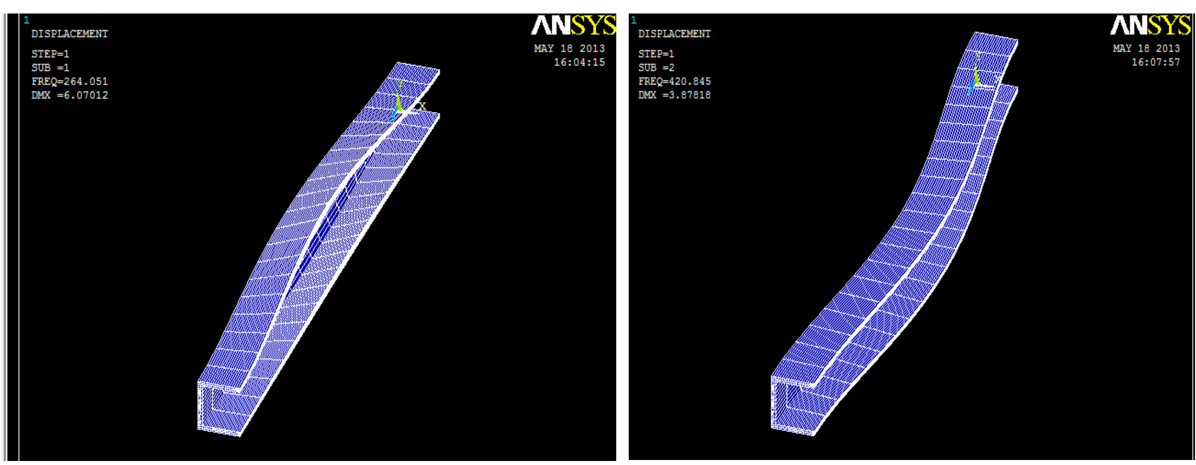


Figure 6.8: Modal analysis of a 8 layer channel beam at fixed-fixed boundary condition by Ansys



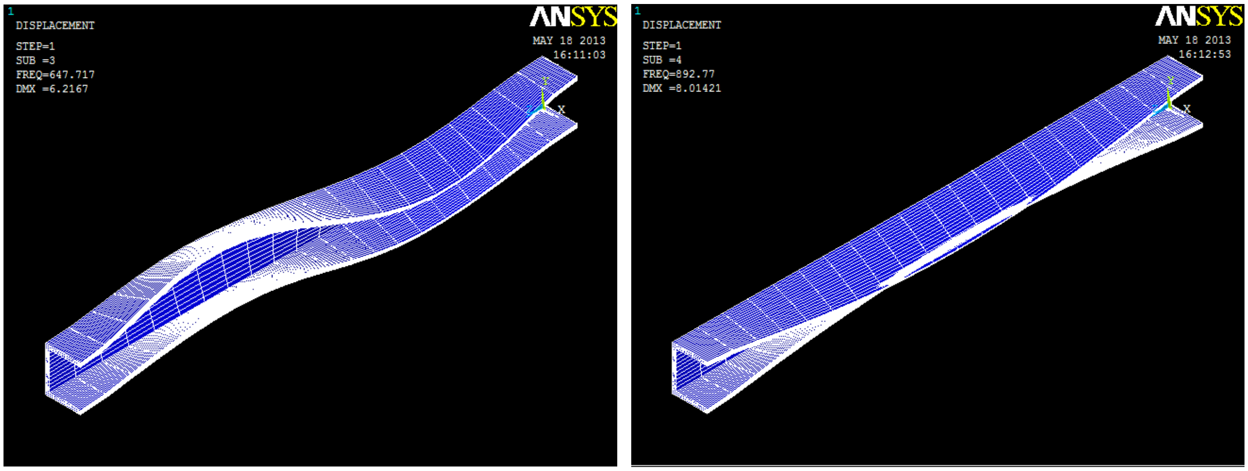


Figure 6.9: Four natural frequency mode shapes of a 8 layer channel beam at fixed-fixed boundary condition by Ansys

Chapter 7

CONCLUSION

CONCLUSION

The following conclusions can be made from the present investigations of the box and channel shaped composite beam finite element. This element is versatile and can be used for static and dynamic analysis of a composite or isotropic beam.

(1) The natural frequencies of different boundary conditions of composite beam have been reported. The program result shows in general a good agreement with the existing literature.

(3) It is found that natural frequency is minimum for clamped –free supported beam and maximum for clamped-clamped supported beam.

(4) Mode shape was plotted for differently supported laminated beam with the help of ANSYS [58] to get exact idea of mode shape. Vibration analysis of laminated composite beam was also done on ANSYS [58] to get natural frequency and same trend of natural frequency was found to be repeated.

(5) There is a good agreement between the experimental and numerical results.

(6) The Finite Element method defined previously is directly applied to the explained examples of generally laminated composite beams to obtain the natural frequencies, the impact of Poisson effect, slender ratio, material anisotropy, shear deformation and boundary conditions on the natural frequencies of the laminated beams are analyzed. And it is found that the present results are in very good agreement with the theoretical results of references.

(7) We assumed different examples and it is found that natural frequencies increase with the value of E1 increases.

(8) It is found that natural frequencies decrease with the increase of beam length.

(9) It is observed that natural frequency increases with increase in number of layers and aspect ratios for both box and channel shaped beams

(10) The material anisotropy has a relatively negligible effect on the mode shapes and the slenderness ratio has considerable effect on all five modes especially on the fifth mode.

7.2 SCOPE OF FUTURE WORK

1. An analytical formulation can be derived for modelling the behaviour of laminated composite beams with integrated piezoelectric sensor and actuator. Analytical solution for active vibration control and suppression of smart laminated composite beams can be found. The governing equation should be based on the first-order shear deformation theory .

2. The dynamic response of an unsymmetrical orthotropic laminated composite beam, subjected to moving loads, can be derived. The study should be including the effects of transverse shear deformation, rotary and higher-order inertia. And also we can provide more number of degree of freedom about 10 to 20 and then should be analyzed by higher order shear deformation theory.

3. The free vibration characteristics of laminated composite cylindrical and spherical shells can be analyzed by the first-order shear deformation theory and a meshless global collocation method based on thin plate spline radial basis function.

4. An algorithm based on the finite element method (FEM) can be developed to study the dynamic response of composite laminated beams subjected to the moving oscillator. The first order shear deformation theory (FSDT) should be assumed for the beam model.

5. The damping behavior of laminated sandwich composite beam inserted with a visco elastic layer can be derived.

6. Static and dynamic stability of composite beams with cracks, delaminations under hygrothermal condition

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